Analytical Modeling of Polling in PLC based AMR Systems

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Abstract. The Poster presents an analytical model of polling in multihop electrical Automatic Meter Reading (AMR) systems. It is based on a Markov Chain and provides explicit formulas for the probability of each state as well as for the recurrence time of the initial state from which the throughput of the system is derived. The Poster includes curves showing the behavior of the system as a function of several parameters such as number of hop-by-hop retransmissions, depth of the multihop scheme, probability of retry, Global Time Out and Reconfiguration time.

Keywords- Markov Chains, Polling, Multihop, AMR, PLC

I. INTRODUCTION

AMR can be accomplished using different communication techniques such as Power Line Communication (PLC) or Wireless. Communication systems which use the power cables themselves as communication medium are more convenient for the utilities and in fact many utilities around the world use their own power cables to reach the meters as well as to communicate with their customers for low rate applications. Narrow band PLC (in contrast with the other technology, broadband PLC), has stable standards in Europe [1,2] since the nineties and it is a mature and low cost technology which may achieve nominal transmission speeds at most up to 9600 bit/s. This type of communication due to the characteristics of power lines imply low throughput rates and high error rate levels [3, 5, 6] that typically mean retry probabilities in the range from 1% to 20%. Another essential characteristic of the power line channel is its tree-like nature with high and complex attenuation and noise patterns that naturally leads to the management of multihop logical structures [6, 7, 8] where a Master station located at the MV/LV transformer substation communicate with meters or customer devices through several intermediate relays.

Creating and managing these structures is a main issue in AMR and since polling is a widely used mechanism in AMR systems for managing them, the present Poster concentrates on polling over these multihop structures providing a model which is applicable to both narrowband as well as broadband PLC and most possibly to Wireless. The model is based on a Markov chain [9, 10] and allows for the explicit calculation of throughput and delay with respect to several system parameters such as number of hops, number of retries and time out values. It provides an in-depth sight into AMR systems which may be of help to researchers on the field. The model includes two versions of procedure upon time-out expiration, one is the typical End-to-End Time-out and the other includes a fix delay to account for the reconfiguration of the tree upon repeated failure of transmission. It has been validated by the help of a Monte Carlo simulation based on OPNET.

Despite of its widespread use, there are only a few analytical studies of polling in AMR and none to our best knowledge focuses on the multihop aspect [3][4].

II. SYSTEM’S ARCHITECTURE AND CHARACTERISTICS

It is assumed, as it is usually the case, that there is a Master polling station connected to the low voltage side of the MV/LV transformer that polls the meters and other customer devices connected to the low voltage network. The meters are assumed to perform two functions, one is as slave station waiting for a poll and the other is as intermediate repeater to reach other meters. These two functions allow for structuring the system as a logical tree with the Master at the root, different levels of branching at the repeaters and with the meters as leaves. In this way a tree-like logical structure somewhat mimicking the physical power line structure is created. The way this tree is created is outside the scope of this poster although the model may help the design of such tree.

III. THE PROTOCOL

The polling protocol being modeled is a typical ARQ mechanism where messages are sent with an error detecting code and with the protection of a timer for retransmission at the hop level (from Repeater/Meter to Meter). The receiver may answer with positive acknowledgments in case of transmission success. This retransmission procedure is repeated as necessary until reaching some maximum retry value. Upon achieving this value the transmitter no longer repeats the message and remains silent waiting for some higher level mechanism to actuate. This mechanism can be of two types: a global Time-Out from the concentrator or a period of time for reconfiguring the tree. The same retransmission scheme is used for the end-to-end (Concentrator to Meter) dialog.
IV. THE MODEL

The polling mechanism is modeled as a two dimension discrete-time Markov chain with states formed by two numbers, the hop count level and the number of effective retransmissions at that level. Figure 1 represents this Markov Chain.

Figure 1. Markov Chain diagram

By simple inspection of the proposed Markov Chain model it should be clear that all states are recurrent since they are continuously visited and every state is reachable from any other state. This means that the chain is irreducible [7].

The mean recurrence time of the initial state is the basic average delay of the system from which other performance measures like throughput can be derived.

The parameters of the Model are:

- \( p \), single try local retransmission probability
- \( t_o \), hop time-out period
- \( t_x \), duration of a basic dialog
- \( r \), maximum number of hop retries
- \( h \), maximum number of or repeater levels
- \( F \), fix time to get the tree reconfigured

- \( T_o \), end-to-end time-out of the concentrator

Since parameters may depend on the link direction being considered (downlink or uplink), we use a letter “d” to indicate downlink direction and letter “u” to indicate uplink.

The hop count level in the Markov Chain is twice the maximum hop count distance between concentrator and meters. The reason is that a complete dialog includes a downlink transmission with a maximum of \( h \) hops and an uplink transmission with \( h \) additional hops.

Each transition between states has associated a transition probability and a delay. All the transition probabilities are a function of the single try local retransmission probability \( p \).

Upon reaching \( r \), two scenarios are considered. One is a global Time-out and the other one is a constant time taking into account the time needed to reconfigure the tree. In Figure 1 only the second one has been represented.

The probability of each state in the Markov Chain is calculated by the use of the classical equilibrium equations for each state and the results are the following:

\[
p(i,j) = p(0,0) \cdot (1 - p \cdot d)^i \cdot p_u ^j \quad \text{for} \quad i \leq h - 1
\]

\[
p(i,j) = p(0,0) \cdot (1 - p \cdot d)^h \cdot p_u ^i \quad \text{for} \quad i = h
\]

\[
p(i,j) = p(0,0) \cdot (1 - p \cdot d)^{i-h} \cdot (1 - p \cdot u)^{(i-h)} \cdot p_u ^j \quad \text{for} \quad i \geq h + 1
\]

To check the accuracy of the state probabilities a simulation study based on OPNET has been carried out.

We are interested in the trajectories leaving and coming to the initial state. Each of these trajectories corresponds to the successful transmission of a message and the model allows for the computation of their delay. Based on the previous probabilities, the average number of visits to states \((i,0)\) made by each trajectory starting and ending in the initial state are calculated and, from them, formulas for the average delay of these trajectories are obtained for the two considered scenarios.

The formula in the scenario with reconfiguration of the tree is given by:

\[
D = d_\text{d} \cdot \frac{1 - p_u \cdot (1 - (1 - p_u)^{r+1})}{1 - p_u \cdot (1 - p_u)^{h+1}} + d_u \cdot \frac{1}{1 - p_u \cdot (1 - p_u)^{h+1}}
\]

The formula for the Global Time out scenario is:

\[
D' = h \cdot (d_\text{d} + d_u') + T_\text{r} \cdot \frac{1}{1 - p_u \cdot (1 - p_u)^{h+1}}
\]

Where \( d' \) and \( d \) are: \( k \) may be \( u \) or \( d \):

\[
d'_k = (1 - p_k^u) \cdot t_{\text{d k}} + t_{\text{u k}} \cdot p_k \cdot \left[ \frac{1 - p_k^{n-1}}{1 - p} \cdot (r_k - 1) \cdot p_k^{n-1} \right]
\]

\[
d_k = d'_k + p_k^u \cdot (t_{\text{d k}} \cdot (r_k - 1) + F_k)
\]

The value of the Time Out that guarantees that only one message will be in the system at a time is equal to the delay of the longest trajectory in the system without returning to initial state and is given by:

\[
T_{\text{max}} = h \cdot (t_{\text{d u}} + t_{\text{u d}}) + h \cdot (t_{\text{d d}} \cdot (r_d - 1) + t_{\text{u d}} \cdot (r_u - 1))
\]

Values below this one may cause the presence of more than one message in the system and, thus, may produce collisions but they may be good to achieve lower average delays.
V. SOME RESULTS

The model developed here has been applied to a system which may be typical of a meter reading situation (one command downlink followed by a longer uplink answer containing the reading of the meter under test).

The parameters have been adjusted to the following values: nominal speed of 4800bit/s, commands of 64 bits of length, returning messages from the meters of 256 bits (payload of 192 bits) and acknowledgments of 32 bits. Delay and throughput as a function of $p$, $r$, $h$, $F$ and $T_o$ have been obtained and some of them are represented in the figures.

![Figure 2. Throughput vs. maximum hop count as a function of the probability of retry](image)

Figure 2 shows that system throughput is almost inversely proportional to the hop-count level. Thus any policy for structuring the tree should take into account hop count as its most important parameter.

Figures 3 and 4 show that the maximum number of retries should be at least 3 to operate efficiently in the range of retry probabilities typical for these systems (1-20%).

![Figure 3. Throughput vs. retry probability as a function of the maximum number of retries](image)

![Figure 4. Throughput vs. maximum number of retries as a function of the retry probability](image)

VI. CONCLUSIONS

A Markov Model of polling in multihop AMR systems has been developed which provides for explicit formulas for the delay and throughput of the system as a function of various system parameters. It can be useful for researchers in the area and can possibly be extended to other communication media besides PLC.

The model opens the door to further insight into AMR systems. For instance one might want to reduce the global Time-out value below the necessary value to avoid collisions. This would improve throughput at the expense of a probability of collision. Also one might explore different tree reconfiguration policies with different delays or extend the model to study the effect of sending an error packet to the Master in case of exhausting the downlink retries.

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References