Connection Admission Control in Cellular Networks: a Discrete Time Optimal Solution

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1. Introduction

- This poster focuses on the Connection Admission Control (CAC), which is a key resource management procedure, and proposes a solution to the CAC problem based on modeling and optimal control methodologies. In particular, it refers to the CAC procedure for an asynchronous Code Division Multiple Access (CDMA) based cellular system.

- The CAC problem is formulated as a discrete time optimal control problem subject to a set of constraints. As a matter of fact, the proposed controller, which solves the CAC problem, computes the control variables so that (i) a set of suitable constraints, which correspond to the QoS requirements (link availability, blocking probability and dropping probability), are respected and (ii) a proper performance index, which evaluates the exploitation degree of the available bandwidth, is maximized.

- In order to obtain the optimal solution, an innovative predictive approach is proposed: in particular the constraints (i) as well as the performance index (ii) are respectively expressed as probabilities and expected values, over a fixed future time interval, conditioned on the past records.

- At each discrete time, the problem is reduced to and solved as an integer-value linear programming one.

- The proposed procedure is successfully tested against suitable simulated data.

- Some validation and comparison is also performed with respect to other existing popular CAC policies.
2. Discrete Time Dynamical Model for Traffic in a Given Cell in its Environment

Let $t_0 < t_1 < \ldots < t_j < \ldots$, be a sequence of discrete times:

- $M_k(t_j | t_i)$: mean value of the number of in progress connections in class $k$, $k \in \{1,2,\ldots,C\}$ at time $t_j$, given all information about traffic available up to time $t_i$, $t_i \leq t_j$.

- $N(t_j)$: the number of stand-by mobile users at time $t_j$.

- $\lambda_k(t_j)$: the connection request rate of any mobile user relevant to class $k$ at time $t_j$.

- $\mu_k(t_j)$: the sum of departure $\mu_k^{(c)}(t_j)$ and termination $\mu_k^{(i)}(t_j)$ rates of any mobile user with a connection in progress relevant to class $k$ at time $t_j$.

- $\lambda_k^{(c)}(t_j)$: the arrival rate of mobile users with a call in progress relevant to the class $k$ from a neighbouring cell at time $t_j$.

- $u_k(t_j) \in \{0,1\}$: **Acceptance Control** for the Service Class $k$ at time $t_j$: if it equals to one, then a new connection relevant to the Service Class $k$ at time $t_j$ is accepted; if it equals to zero, then the same is rejected.

- $v_k(t_j) \in \{0,1,\ldots,M_k(t_j)\}$: **Dropping Control** for the Service Class $k$ at time $t_j$: specifically, $v_k(t_j)$ denotes the number of connections relevant to the Service Class $k$ which have to be forcibly dropped at time $t_j$.

Dynamic Equations:

- $M_k(t_{j+1} | t_i) = M_k(t_i) - v_k(t_i) - M_k(t_i)\mu_k(t_i)(t_{j+1} - t_j) + (\lambda_k(t_i)N(t_i)u_k(t_i)) + \lambda_k^{(c)}(t_i)(t_{j+1} - t_j)$;

- $k = 1,2,\ldots,C$; $i = 0,1,\ldots$. 

- $B$ Base Stations
- $M$ Mobile Terminals
3. QoS Constraints and Performance Index (1/2)

- Assume $p_k(t_{i+1})$ is the ratio between the power density received by the base station from a class $k$ connection at time $t_{i+1}$ and the useful energy per bit received by the base station itself. A simple forecast model provides the following relationship:

$$p_k(t_{i+1} \mid t_i) = \frac{\sum_{j=0}^{i} e^{-\theta(t_{i+1} - t_j)} p_k(t_j)}{\sum_{j=0}^{i} e^{-\theta(t_{i+1} - t_j)}} \quad k = 1, 2, \ldots, C$$

where $p_k(t_{i+1} \mid t_i)$ appears as a convex combination of the known past values $p_k(t_j), j \leq i$, via exponentially growing coefficients with a suitably chosen discounting rate $\theta_k$.

- Let

$$a_k(t_i) = \lambda_k(t_i)(t_{i+1} - t_i)p_k(t_{i+1} \mid t_i) \quad b_k(t_i) = p_k(t_{i+1} \mid t_i)$$

$$c(t_i) = \eta(t_i) - \sum_{k=1}^{c} \left[ M_k(t_i)(1 - \mu_k(t_i)(t_{i+1} - t_i)) + \lambda_k^{(e)}(t_i)(t_{i+1} - t_i)p_k(t_{i+1} \mid t_i) \right]$$

$$\delta_k(t_i) = \rho_{1k} + \sum_{j=0}^{i-1} (\rho_{ik} - u_k(t_j))(n_k(t_{j+1}) - n_k(t_j))$$

$$\zeta_k(t_i) = \frac{\rho_{2k}}{1 - \rho_{2k}} \left[ m_k(t_i) + \mu_k(t_i)M_k(t_i)(t_{i+1} - t_i) \right] - \sum_{j=0}^{i-1} v_k(t_j)$$

be appropriate coefficients, depending on the effective past traffic conditions, where $\eta(t_i)$ represents a threshold which possibly accounts for the statistics of external and thermal power densities; $\rho_{1k}$ and $\rho_{2k}$ are respectively the fixed threshold fraction of blocked and dropped traffic; $n_k(t_i)$ and $m_k(t_i)$ are respectively the known number of requested class $k$ connections and the known number of departed and spontaneously terminated class $k$ connections up to $t_i$. 
4. QoS Constraints and Performance Index (2/2)

- **Link Availability constraint**: the ratio between the total power density received by the base station and the useful energy per bit received by the base station itself has to be lower than a given threshold over a sufficiently large percentage of time in the interval \([t_i, t_{i+1})\):

\[
N(t_i) \sum_{k=1}^{C} a_k(t_i) u_k(t_i) - \sum_{k=1}^{C} b_k(t_i) v_k(t_i) - c(t_i) \leq 0
\]

- **Blocking Probability constraint**: the mean value of the accepted calls for each class \(k\) up to time \(t_{i+1}\), conditioned on all traffic information available up to \(t_i\) has to be not lower than a fixed fraction of the mean value of the total number of requested calls up to \(t_{i+1}\) conditioned on the same information:

\[
u_k(t_i) \geq \delta_k(t_i) \quad k = 1, 2, ..., C
\]

- **Dropping Probability constraint**: for each class the number of forcedly dropped calls up to time \(t_{i+1}\) has to be not greater than a fixed fraction of the mean value of the total number of terminated calls conditioned on all traffic information available up to time \(t_i\):

\[
v_k(t_i) \leq \zeta_k(t_i) \quad k = 1, 2, ..., C
\]

- **Performance Index**: the expected value of the total throughput at time \(t_{i+1}\), conditioned on all traffic information available up to time \(t_i\), has to be maximized with respect to the control variables \(u_k(t_i), v_k(t_i), k = 1, 2, ..., C\):

\[
J(t_i, u_1(t_i), ..., u_C(t_i), v_1(t_i), ..., v_C(t_i)) = N(t_i) \sum_{k=1}^{C} w_k a_k(t_i) u_k(t_i) - \sum_{k=1}^{C} w_k b_k(t_i) v_k(t_i)
\]

where \(w_k\) are suitable non negative weights.
5. CAC Problem Solution

1. Mathematical Structure of the CAC problem: the CAC problem has been reduced to a sequence of classical integer valued linear programming problems, usually known as Knapsack Problems.

2. Necessary and Sufficient Existence Conditions (NSC): being the admissible control set finite, an optimal solution exists whenever the set itself is non empty. A necessary and sufficient set of conditions for that is easily obtained for each $t_i$:

$$\delta_k(t_i) \leq 1 \quad \zeta_k(t_i) \geq 0 \quad k = 1, 2, \ldots, C$$

$$N(t_i) \sum_{k=1}^{C} a_k(t_i) \left[ \delta_k(t_i) \right] - \sum_{k=1}^{C} b_k(t_i) \min \{ \zeta_k(t_i) \} M_k(t_i) \leq c(t_i)$$

where $\lfloor x \rfloor$ denotes the least non negative integer not less than $x$ and $\{ x \}$ denotes the integer part of $\max(0, x)$.

3. Computation of the Solution:
   a. If the NSC are verified the optimal solution is computed by exploiting a standard routine for Knapsack problems (C-PLEX module).
   b. Conversely, if one of NSC is violated, then obviously no admissible control exists since one of the constraints would be violated as well (for the sub-problem corresponding to time $t_i$). So a relaxed sub-problem is considered (for the same time $t_i$) and the corresponding solution is computed.
6. Results and comparisons

Simulation Scenario: Several different traffic scenarios have been simulated and the proposed Optimal control procedure has been applied with satisfactory results. Also a comparison with other two control procedures (Arrows and Interference) are reported. We report the results in the following case: Fixed Background data (ftp + mail) traffic (150 Kbps) and variable conversational traffic (C=2).

\[ \eta(t_i) = \eta = 4.07 \]
\[ \theta_1 = 60 \text{ sec} \]
\[ \theta_2 = 2.4 \text{ sec} \]
\[ \rho_{11} = \rho_{12} = \rho_1 = 0.95 \]
\[ \rho_{21} = \rho_{22} = \rho_2 = 0.02 \]
\[ t_f - t_0 = 1 \text{ hour} \]
\[ t_{i+1} - t_i = 20 \text{ ms} \]

1. Optimal CAC
2. Arrows CAC
3. Interference CAC
4. A posteriori Link Availability
5. A posteriori Performance Evaluation
6. Optimality Default
7. Conclusions

- It appears from Fig.s 1-4 that for the fixed background data traffic (150 kbps) the ARROWS CAC and the Interference CAC allow the satisfaction of QoS constraints, assessed as a posteriori frequency constraints, up to a conversational traffic of 30 Erl and 34 Erl respectively, while the proposed Optimal CAC achieves the limit of about 60 Erl.

- From Fig. 5 it appears that, as far as the total throughput is concerned, the three CAC's behave in a similar way as long as the QoS constraints are satisfied (~30-35 Erl of conversational traffic). It is of paramount importance that the Optimal CAC achieves a significant further improvement in the range up to 60 Erl.

- The performance results of the proposed Optimal CAC appear widely better than the two other ones because the optimal CAC policy exploits at the best the degree of freedom offered by the QoS constraints (as we can see in fig 4). In addition, in order to satisfy the QoS constraints, the traffic available information is exploited to obtain a forecast on the future traffic trend.

- The figure 6 shows the percentage of instants in which the NSC of existence for the optimal solution are violated. It can be noted that this percentage is negligible until a conversational traffic of 60 Erl.
7. References


