Reachability: An Alternative to Connectivity for Sparse Wireless Multi-hop Networks

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***Abstract*— Connectivity has been widely used as a measure of the extent to which nodes can communicate in a wireless multi-hop network. We claim that connectivity, defined as the probability that the network forms a single connected component, is not an accurate measure of the communication capabilities of the network, particularly when the network is sparse. We claim that the fraction of connected node pairs, a term we call reachability, is a more appropriate measure in such networks. We illustrate this using simulation results. We also outline an empirical characterization of reachability using regression analysis.**

I. Introduction

A fundamental limiting factor in a wireless multi-hop network is the absence or presence of routes between nodes. It is beyond this that factors like channel capacity and interference can affect the extent of communication. The most popular measure for determining the extent to which a wireless multi-hop network is connected is the probability that the network graph forms a single connected component. This probability is called *connectivity*, and has been extensively used in literature to measure the communication capabilities of wireless multi-hop networks.

A *sparse* wireless multi-hop network is one in which connectivity with high probability is not ensured. Such a network can arise in various ways: a vehicular ad hoc network in an area with low traffic density, an initially connected sensor network after some of its nodes have failed, and an ad hoc communications network that is being deployed incrementally can all be sparse networks. Occasionally, in a constrained deployment scenario, we may even wish to deploy a multi-hop network that trades off connectivity for cost. In such sparse networks, using connectivity as a metric or design parameter can prove inadequate because i) connectivity is not indicative of the actual extent to which the network can support communication; and ii) it is unresponsive to fine changes in network parameters. For example, it is possible that a sparse network that allows a significant number of nodes to communicate has a probability of connectivity close to zero. Further, an increase in some network parameter such as number of nodes, or transmission range, may increase the ability of nodes to communicate, but it may not be reflected by a corresponding increase in connectivity. We believe that a property of the network graph better suited for use with sparse networks is the *fraction of node pairs that are connected*. We call this quantity *reachability*. We consider both connectivity and reachability to be different *connectivity measures* of a network graph.

II. How Connectivity can be Misleading

The poster will illustrate the pitfalls of using connectivity with sparse networks through graphs comparing the growth of reachability and connectivity for different kinds of sparse networks.

Figure 1 is obtained from simulations, and plots the growth of reachability and connectivity as the uniform transmission range of nodes, R, increases for 60 static nodes distributed uniformly at random in a 2000m $\times$ 2000m area. Note that in this case, when reachability is 0.4, meaning 40% of node pairs are connected, connectivity is still at zero. Further, using only connectivity here is clearly inappropriate since the connectivity curve would lead us to believe that increasing R from 50 to any value less than 320 would have no effect on the extent of communication supported by the network.

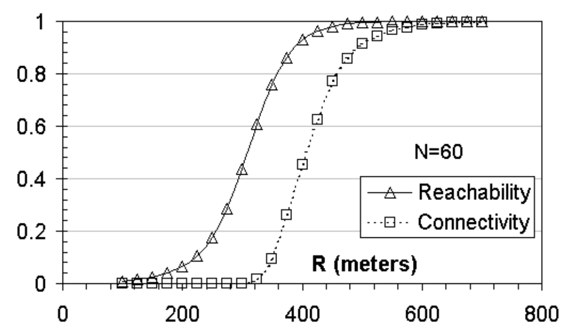

Fig. 1. Increasing R, no mobility

Similar conclusions can be drawn from Fig. 2 which plots the increase of reachability and connectivity as the number of nodes, N, increases with R kept constant at 300m.

Sparse networks frequently use their mobility to compensate for low connectivity. Figure 3 shows one such scenario where nodes were made to move with a uniform velocity of $5ms^{-1}$. Note the increased difference between connectivity and reachability from Fig. 2.

Sparse networks also use mobility coupled with asynchronous communication as in store-and-forward routing mechanisms. Nodes can buffer packets meant for other nodes and deliver them to the intended destination if it comes within range, or the node can pass it on to some other node for

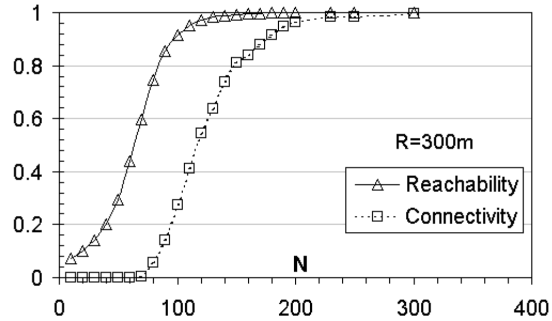

Fig. 2. Increasing N, no mobility

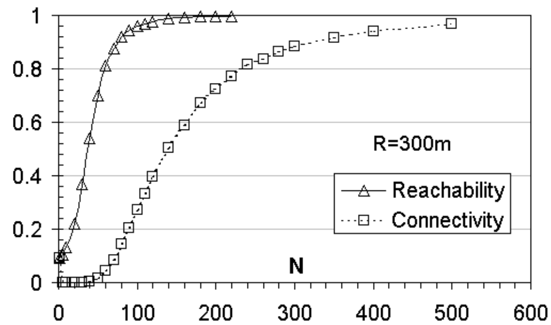

Fig. 3. Increasing N, with mobility

buffering. In the presence of a scheme like this, there is an even greater disparity between reachability and connectivity. In the network of Fig. 1 uniform mobility of $5ms^{-1}$ was introduced, and nodes were considered connected at a time instant if there existed a possibly asynchronous path between them within 30 seconds from that instant. Note that reachability is at 0.8, meaning 80% of the nodes can communicate, when connectivity has not yet risen from zero.

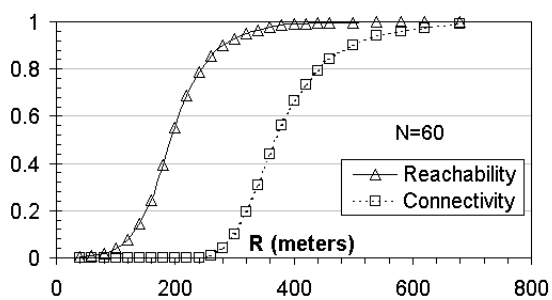

Fig. 4. Increasing R, with mobility and asynchronous communication

III. Context

The ability to evaluate tradeoffs between deployment parameters is important in wireless multi-hop networks. Gupta and Kumar showed in [1] how throughput per source-destination pair in a wireless multi-hop network decreases as node density increases. Grossglauser and Tse in [2] later showed that mobility could be exploited to achieve a tradeoff between throughput and delay. This would allow throughput to be maintained almost constant even with increasing node density. Similarly, a tradeoff has also been achieved between *connectivity* and delay. Delay tolerant routing [3] and Message Ferrying [4] are representative of work that uses node mobility to achieve asynchronous communication between disconnected nodes in sparse networks. Connectivity is also a limiting factor in sparse networks: in the absence of paths between nodes, issues of interference and network capacity become irrelevant. This motivates the study of appropriate metrics that allow fine grained tradeoffs.

IV. Reachability

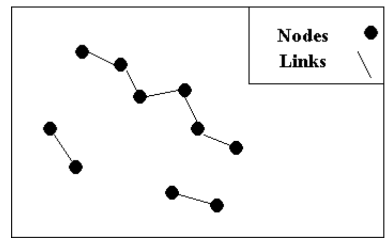

Fig. 5. A network instance with reachability = 0.378

The reachability of a static network is defined as the *fraction of connected node pairs* in the network. It is a property of the network graph, with no assumptions made regarding the distribution of nodes. Using this definition we can calculate reachability for a network of *N* nodes as:

$$Reachability = \frac{\text{No. of connected node pairs}}{\binom{N}{2}} \qquad (1)$$

A pair of nodes is considered connected if there is a path of length one or greater between them. Figure 5 shows one instance of a small network with 10 nodes. We count the number of node pairs that can reach each other, that is, nodes that are connected either directly or through other nodes, as 17. Substituting $N = 10$ in the denominator of Eqn. 1, we obtain the reachability for this network instance as $17/45$ or 0.378.

V. Characterizing Reachability

Our network model is as follows: *N* nodes, each with a transmission range of *R* are distributed uniformly at random in a square area of side l; $r = R/l$ is the normalized transmission range, and *M* denotes the mobility parameters. We denote the value of reachability for such a network as $Rch^{M}_{N,r}$. In the static case, we represent it as $Rch_{N,r}$. If the *N* nodes form *k* components with m_i nodes in the i^{th} component, we can rewrite Eqn. 1 as

$$Rch_{N,r} = \frac{\sum_{i=1}^{k} \binom{m_i}{2}}{\binom{N}{2}} = \frac{\sum_{i=1}^{k} m_i(m_i-1)}{N(N-1)} \qquad (2)$$

It may be possible to obtain asymptotic bounds for $Rch_{N,r}$, but since sparse networks often involve small numbers of nodes, we are particularly interested in characterizations in the finite domain, and chose to model reachability through empirical regression.

We explored data from comprehensive simulations, and found that $Rch_{N,r}$ obeys logistic growth as given by:

$$Rch_{N,r} = \frac{1}{1 + e^{\alpha_N - \beta_N r}} \tag{3}$$

The logistic curve is most often used to model the growth of populations in biology. It represents slow initial growth from zero followed by a rapid growth, and then a slow growth to 1. Figure 6 show the close agreement of simulated data and the corresponding logistic curve.

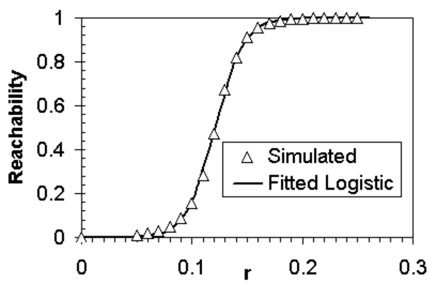

Fig. 6. Logistic fit for N=100

We then:

- conducted extensive simulations to obtain data that represented the growth of $Rch_{N,r}$ from 0 to 1 as *r* increased, while keeping *N* fixed;
- used Eqn. 3 as a regression function for simulated data, and obtained the coefficients α and β for the corresponding value of *N*—this allowed us to characterize reachability as a function of *r* for one value of *N*;
- We repeated the above two steps for values of *N* ranging from 2 to 500, and performed a second level of regression on the estimated values of α_N and β_N.

This gave us equations that along with Eqn. 3 allow us to obtain reachability as a function of *N* and *r* for values of *N* ranging from 2 to 500.

$$\begin{aligned}\alpha_N \ &= 3.815(1 - e^{-4.091\times10^{-2}N}) \\ &+15.4(1 - e^{-2.055\times10^{-3}N}) \\ &+3.004 \qquad 2 \le N \le 500\end{aligned} \tag{4}$$

$$\begin{aligned}\beta_N \ &= 5.141 + 0.9421N \\ &-2.597\times10^{-3}N^2 + 8.42\times10^{-6}N^3 \\ &-1.37\times10^{-8}N^4 + 1.058\times10^{-11}N^5 \\ &-3.209\times10^{-15}N^6 \qquad 2 \le N \le 500\end{aligned} \tag{5}$$

On validating the model, we found that the average relative error in the predicted $Rch_{N,r}$ was 3.5%. We did not observe a single instance when the model was in error by more than 0.05. We have also extended this model to make it usable up to $N = 1000$, but with a larger margin of error. A detailed account of the characterization can be found in [5].

VI. Tools using Reachability

Simran[1] is a simulator we have developed for studying topological properties of wireless multi-hop networks. Simran takes as input a scenario file with initial positions and movement scripts of nodes, and generates a trace file containing metrics of interest such as average number of neighbors, averaged shortest path lengths over all pairs of nodes, reachability, connectivity, and number and size of connected components. All simulations for generating the graphs in this proposal were conducted using Simran. Spanner[2] is a design tool that uses the reachability model described in Sec. V. Given three values from deployment area, N, R, and reachability, it computes the fourth. These tools can be used to evaluate design tradeoffs in sparse wireless multi-hop networks.

VII. Conclusions

Sparse networks are capable of supporting a significant extent of communication. This is specially true in the presence of mobility coupled with asynchronous communication, as seen in Fig. 4. However, using connectivity to measure the extent of communication possible in such a network can be misleading. We presented a metric useful in such networks, reachability, and outlined its empirical characterization. A case study where reachability is used for facilitating tradeoffs in deployment of sparse networks can be found in [6].

References

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